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Long-distance properties of frozen U(1) Higgs and axially U(1)-gauged four-Fermi models in $1+1$ dimensions

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Abstract

We study the long-distance relevance of vortices (instantons) in an N-component axially U(1)-gauged four-Fermi theory in $1+1$ dimensions, in which a naive use of $1/N$ expansion predicts the dynamical Higgs phenomenon. Its general effective lagrangian is found to be a frozen U(1) Higgs model with the gauge-field mass term proportional to an anomaly parameter (b). The dual-transformed versions of the effective theory are represented by sine-Gordon systems and recursion-relation analyses are performed. The results suggest that in the gauge-invariant scheme ($b=0$) vortices are always relevant at long distances, while in non-invariant schemes ($b>0$) there exists a critical N above which the long-distance behavior is dominated by a free massless scalar field.

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1 Introduction

In a previous paper [1] we have stu
of the quasi (2+1)-dimensional four-F
metry, in order to get insight into the
superconductors. Remarkably we fou
Kosterlitz-Thouless (KT) type and ar
It is now a natural question to ask wh
the model is coupled to an axial gaug
This question may be well parallel to
namely, if a Meissner effect takes plac
applied to quasi two-dimensional supe
that the qualitative long-distance prop
temperature is essentially similar to t
temperature, in which a local gauge sy

On the other hand, four-Fermi inte
tentions in particle physics as a sour
Standard model [2]. In this context to
Fermi interaction and condensates to
main ingredient of these arguments
nism [3] with a mean-field-type appro
of field-theoretical aspects have recentl
those for gauged models including top
In 3+1 dimensions it is not straightfo
effect of topological excitations and o
lattice. In such situations it would be
models with an abelian gauge symme
tractable. Although results themselve
ity and an abelian nature of the gaug
and field-theoretical lessons which ma
higher-dimensional models with (non-

In view of these backgrounds, we w
non-perturbative long-distance prope

Fermi theory coupled to an axial gauge field in $1+1$ dimensions [7], as a simple model to discuss the dynamical Higgs phenomenon. In standard notations the model is defined with the lagrangian

$$\mathcal{L} = \bar{\psi}\cdot(i\partial\!\!\!/ - \sigma - i\gamma_5\pi + A\!\!\!/\gamma_5)\psi - (N/2g^2)(\sigma^2+\pi^2) - (N/4e^2)F^2, \qquad (1.1)$$

where g and e are respectively a dimensionless four-Fermi coupling and an axial-vector gauge coupling with a mass dimension, and a dot $(\cdot)$ means to take a sum over fermion species $i = 1 \sim N$ (Lorentz indices are suppressed). $\mathcal{L}$ is invariant under the following axial gauge transformation:

$$\begin{array}{rcl} \psi(x) & \rightarrow & \mathrm{e}^{i\gamma_5\varphi(x)}\psi(x), \\ A(x) & \rightarrow & A(x) + \partial\varphi, \\ (\sigma+i\pi)(x) & \rightarrow & \mathrm{e}^{-2i\varphi(x)}(\sigma+i\pi)(x). \end{array} \qquad (1.2)$$

In their classic paper [7], Gross and Neveu studied the limit $N \to \infty$ of (1.1) for fixed g^2 and e^2. For $A = 0$, $\mathcal{L}$ is invariant under scale and global chiral transformations. In the limit $N \to \infty$ the dynamical breaking of chiral symmetry develops for all non-zero g^2, and σ acquires a vacuum expectation value (v.e.v.) $<\sigma>=\sigma_{\mathrm{c}}$, giving a mass to the fermions (dimensional transmutation) [7]. In $1+1$ dimensions, however, this symmetry-broken solution, which we call a *NG vacuum*, is unstable against higher-order corrections of $1/N$ expansion. Beyond the leading order, uncontrollable infrared divergences appear in the gap equations (cancellation conditions of σ tadpoles: $<\hat{\sigma}>=0$ where $\hat{\sigma} \equiv \sigma - \sigma_{\mathrm{c}}$) due to the fluctuations of a massless NG boson π (fig.1) and render the leading-order solution meaningless for a general finite N [8]. The absence of a stable NG vacuum is in accord with the general no-go theorem [9]. The meaningful solution for the non-gauged model was later given by Witten [10], who argued that a stable solution is provided by assuming a v.e.v. only for a modulus field ρ ($<\rho>=\rho_{\mathrm{c}}$) of the order-parameter field

$$\sigma + i\pi = \rho\,\mathrm{e}^{i\chi}, \qquad (1.3)$$

but not for a phase field χ. At large N the long-distance behavior is described mainly by a free massless phase field, i.e. with the effective lagrangian

$$\mathcal{L}_\chi = (N/8\pi)(\partial\chi)^2. \qquad (1.4)$$

Other terms for χ (only) are all of hig
distances. The chiral U(1) symmetry
KT phase [11, 12]. It is then expecte
(KT boson) in higher-order diagrams
singularity but only supplies finite re
effective lagrangian (1.4). The abse
explicitly in the second-order gap equ
and would only provide finite renorma

Thus the vacuum specified by

$$\rho_{\mathrm{c}} \neq 0,$$

which we call a *KT vacuum*, is expect
N the long-distance behavior should
temperature phase of the XY model.
power-law decay of a phase-wave corr

$$< \mathrm{e}^{-i\chi(x)}\mathrm{e}^{i\chi(0)} >$$

where const. stand for some positive c

In ref. [7] Gross and Neveu have als
axial gauge coupling ($A \neq 0$). Based
the two-point effective action for A
showed that both of their propagators
the dynamical Higgs mechanism. As a
screened. We should note that, in con
field argument in the gauged model is
$1+1$ dimensions and hence its picture
large fluctuations, whatever the dimen
NG particle generating an infrared sin

Then we would like to ask, to wh
phenomenon is correct. A possible so
from *topological excitations* missed in
in dilute-instanton-gas calculations [2
ious models usually work toward diso
naive mean-field argument *if they are*

Another point that should be considered carefully is a subtlety with respect to an axial-gauge invariance, which is also neglected in the old argument [7]. An anomaly could generally emerge if the fermions are integrated with a gauge non-invariant regularization. In $1+1$ dimensions, however, this reflection manifest itself on the effective action just as a contact term A^2 of the axial gauge field [13, 14, 15] and does not bring out a theoretical difficulty; a massive abelian gauge (Proca) theory is quantum-mechanically consistent. Hence, following recent studies of anomalous gauge theories [16, 17], it is natural to consider that the quantum theory of such a type possesses a hidden one-parameter degree of freedom which cannot be uniquely determined from the theoretical consistency. Once such a mass term is allowed it should play an important role in the long-distance physics.

We thus believe it important to study whether the naive argument in favor of a dynamical Higgs mechanism could persist to hold or not even after incorporating above two points.

In $(1+1)$-dimensional models with an abelian symmetry, the relevant topological excitation is a vortex configuration in a euclidean space-time (instanton). As is suggested from our previous work [1], in order to incorporate vortex configurations in the present four-Fermi theory we should start from the KT vacuum (1.5) keeping a global chiral U(1) symmetry. This can conveniently be achieved with a radial parametrization of the order-parameter field (1.3) [10, 1]. In sect.2 we will show that a general large-N long-distance effective lagrangian on the KT vacuum consists of a U(1) Higgs model with a radial fluctuation frozen and of the possible gauge-field mass term, the coefficient ($\equiv b$) of which is arbitrary. We will thus study the long-distance properties of the effective lagrangian in two cases separately, i.e. in a gauge-invariant scheme ($b=0$, sect.3) and in non-invariant (anomalous) schemes ($b>0$, sect.4).

The (normal) U(1) Higgs model itself is an interesting physical system, for example as effective theories of thin-film superconductors or a helium superfluid threatened by a magnetic field, which in fact contain vortex solutions and some early analyses have been reported [18]. In our knowledges, however, a proper quantum-mechanical treatment of the gauge-vortex dynamics has not yet appeared in references. With the help of a dual transformation, we will develop in sect.3 a renormalization-group (RG) analysis. We first formulate the partition function of

the frozen U(1) Higgs model with an e
system is then equivalent with that o
scalar field. This scalar-vortex system
vortex chemical-potential controlled by
a large chemical potential ($y \ll 1$) ca
Gordon (s-G) field theory. In this loca
effects among vortices using a recursio
the analysis is restricted to a small-fu
an important suggestion about the ph
in particular whether there could exist
the mean-field treatment is justified a

Then in sect.4 we will study the ge
non-invariant scheme, i.e. with a non-
procedures to the above we will sho
vortices interacting with both massive
a large chemical potential ($y \ll 1$) is
and massive s-G fields and the recursio

Sect.5 is devoted to summary and
some technical details for deriving m
pendix B is presented an approximate
for the original $y=1$ system in the b

2 Effective lagrangian
U(1)-gauged four-Fe

In this section, based on the KT v
gauged four-Fermi model (1.1) the lar
responding to (1.4) for the non-gauge
vide a basis on which we can later inve
configurations.

In terms of the radial parametriza

four-Fermi system we will consider, is described by the partition function

$$Z = \int \rho\mathcal{D}\rho\mathcal{D}\chi\mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi\, \exp\left\{i\int d^2x\mathcal{L}'\right\}, \tag{2.1}$$

where a local gauge symmetry is not yet fixed. In (2.1), shifting a modulus field as $\rho + \rho_{\rm c}$, we have

$$\begin{aligned} \mathcal{L}' \;&=\; \bar{\psi}\cdot(i\not{\partial} - \rho_{\rm c} + \not{A}\gamma_5)\psi - (N/4e^2)F^2 - N\rho_{\rm c}\,\rho - N\rho^2/2 \\ &\quad -\rho_{\rm c}\bar{\psi}\cdot({\rm e}^{i\chi\gamma_5} - 1)\,\psi - \rho\,\bar{\psi}\cdot{\rm e}^{i\chi\gamma_5}\,\psi \,+\, {\rm const.}. \end{aligned} \tag{2.2}$$

To the leading order of $1/N$ expansion, a gap equation (a cancellation condition of ρ tadpoles $<\rho>= 0$, corresponding to the diagrams in fig.3) reads

$$0 = \frac{\rho_{\rm c}}{g^2} - i\int\frac{d^2k}{(2\pi)^2}\,{\rm Tr}\,\frac{1}{\not{k} - \rho_{\rm c}}. \tag{2.3}$$

With an ultraviolet cut-off scale Λ, the lowest-lying-energy solution is given by

$$\rho_{\rm c} = {\rm e}^{-\pi/g^2}\Lambda = {\rm e}^{-\pi/g_R^2(\mu)}\mu, \tag{2.4}$$

where a renormalized coupling g_R^2 is defined by

$$g^{-2} = g_R^{-2}(\mu) + (2\pi)^{-1}\ln(\Lambda^2/\mu^2) \tag{2.5}$$

with μ a renormalization mass scale.

For all positive region of g^2 the gap $\rho_{\rm c}$ is positive and gives fermions a RG-invariant mass satisfying a homogeneous RG equation

$$\left[\,\mu(\partial/\partial\mu) + \beta(g_R^2)(\partial/\partial g_R^2)\,\right]\rho_{\rm c} = 0, \tag{2.6}$$

with a β function

$$\beta(g_R^2) = \mu(\partial/\partial\mu)g_R^2(\mu) = -(1/\pi)(g_R^2)^2. \tag{2.7}$$

Note that as is in the non-gauged model [10], even if fermions acquire a mass through a non-zero $\rho_{\rm c}$ the global chiral symmetry of full $\mathcal{L}'$ is kept unbroken. In other words, the present KT scheme of the four-Fermi dynamics drives a dimensional transmutation but not a spontaneous breaking of the global chiral symmetry.

Integrating over fermions we obtai
$1/N$. The long-distance part is contai
including their mixing (fig.4). They re

$$S_2 = N\int\frac{d^2p}{(2\pi)^2}$$

with

$$\begin{aligned} \mathcal{L}_{\rm inv.}(p) \;&=\; (2e^2)^{-1}A^\mu(-p \\ &\quad +(2\pi)^{-1}\rho_{\rm c}^2U\,[\\ \mathcal{L}_{\rm anom.}(p) \;&=\; (2\pi)^{-1}\,b\,A^\mu(- \end{aligned}$$

where $A^\mu(p)$ and $\chi(p)$ represent Fourie
Here U and V are functions of p^2 and

$$\begin{aligned} U(p^2, \rho_{\rm c}^2) \;&=\; \frac{2}{\sqrt{p^2(p^2-4}} \\ V(p^2, \rho_{\rm c}^2) \;&=\; (\rho_{\rm c}^2U - 1)/ \end{aligned}$$

They have the following well-defined de

$$\begin{aligned} \rho_{\rm c}^2U(-\partial^2, \rho_{\rm c}^2) \;&=\; 1 \\ V(-\partial^2, \rho_{\rm c}^2) \;&=\; (\end{aligned}$$

accompanied by inverse powers of (ma
consists of two parts: one is a locally
other is a non-invariant contact mass
in the evaluation of a local contribu
anomaly [13, 14, 15]. In (2.10) a param
ter in the literatures of anomalous gau
depending on a regularization procedu

Since an ultraviolet regularization
polarization diagram (fig.4(a)), it is su
lagrangian $\mathcal{L}'$ the following Pauli-Villa

$$\mathcal{L}_{\rm PV} = \bar{\psi}_1^{\rm PV}\cdot(\,i\not{\partial} + b_1\not{A} + \Lambda\,)\,\psi_1^{\rm PV}$$

with $b_1^2+b_2^2=1$, the condition required for the cancellation of apparent logarithmic divergences. Of course, other regularization prescriptions such as a point-splitting method would also be available. In the present Pauli-Villars method, starting from $\mathcal{L}'+\mathcal{L}_{\mathrm{PV}}$, we in fact obtain (2.8) with b in (2.10) given explicitly by

$$b=-1+b_1^2-b_2^2=2\,(b_1^2-1), \tag{2.14}$$

which is an arbitrary constant. [2]

$\mathcal{L}_{\mathrm{inv.}}$ in (2.9) is (axially) gauge invariant and one should get only this part provided he imposes the gauge invariance on the gauge-field vacuum polarization by employing, for example, the above Pauli-Villars method with the special choice $b_1=1\,(b_2=0)$. Alternatively, for the practical purpose this gauge-invariant result is most easily achieved by use of the dimensional regularization, defined only in momentum integrals with all algebras of γ^μ matrices unchanged from those of two-dimensional ones (a dimensional reduction) [20], or that with an assumption that γ_5 anticommutes with all γ^μ matrices in continuous d dimensions [21]. In this gauge-invariant system (2.9), if χ is single valued, we can absorb it into a gauge-invariant vector field

$$B^\mu(p)=A^\mu(p)+(ip^\mu/2)\chi(p), \tag{2.15}$$

and a χ integration is decoupled from the theory. Then, eq.(2.9) is reduced to a free part of the neutral massive vector theory with the propagator $D_{\mu\nu}$

$$iD_{\mu\nu}(p)=e^2(g_{\mu\nu}-(p_\mu p_\nu/p^2))(p^2-\alpha)^{-1}-(\pi/\rho_{\mathrm{c}}^2U)(p_\mu p_\nu/p^2), \tag{2.16}$$

which possesses a pole at $p^2=\alpha$. This mass $\alpha^{1/2}$ is independent of the four-Fermi coupling g^2 and is identical with that of the Schwinger model [19]. Although our effective action (2.8) is gauge invariant the feature of the spectrum is the same as that predicted in the broken-symmetry ($\sigma_{\mathrm{c}}\neq 0$) argument [7]. This may thus be called a gauge-invariant version of the dynamical Higgs mechanism in a (1+1)-dimensional chiral four-Fermi theory.

[2] Although the hermiticity of $\mathcal{L}_{\mathrm{PV}}$ is lost except for $0\le b_1^2\le 1$, it would not lead to a serious matter because ψ_i^{PV} are introduced only for a technical purpose, i.e. only as regulator fields and disappear from the physical spectrum ($\Lambda\to\infty$) after fermion-loop computations. The remnant of the regulators only emerge as $\mathcal{L}_{\mathrm{anom.}}$.

As is mentioned above, although b c
ance, we are in principle free to choos
Since the massive abelian gauge theor
tem, any choice of $b(\geq 0)$ would give a
arbitrary, we therefore consider our eff
of theories, and will henceforth study
This idea of the hidden-parameter gene
of anomalous gauge theories [16, 17].

To the leading order the two-poin
use of the gap equation (2.3). The ρ f
a mass $2\rho_{\mathrm{c}}$. In higher orders the integr
give finite corrections of $\mathrm{O}(N^0)$ to $\mathcal{L}_{\mathrm{in}}$

Even for the gauge-invariant syster
the above Higgs picture if χ is *not* si
F of B will induce singularities and
Higgs mechanism based on the replac
possibility can generally happen since
((1.2), (1.3)) and need not be single v
physically realized as topological excit
present abelian theory in $1+1$ dimens
distances, it is useful to extract the lo
gauge-invariant part $\mathcal{L}_{\mathrm{inv.}}$. This effecti
gauge-invariant local operator F and
with the following lagrangian:

$\mathcal{L}_{\mathrm{Hig.}}=-(\beta/4$

where the gauge field has been rescale

$\beta\ \equiv\ (N/4e_{\mathrm{eff.}}^2)=(N/4$

$\kappa\ \equiv\ (N/4\pi)\,[1+\mathrm{O}(1$

$\mathcal{L}_{\mathrm{Hig.}}$ is nothing but a U(1) Higgs m
continuum version of the XY model co

same local gauge symmetry as the non-local action $\mathcal{L}_{\text{inv.}}$ and contains its lowest-derivative part for the gauge and Higgs dynamics. The higher-derivative terms which have been neglected include $(\partial F)^2$, $\partial^2|D\Psi|^2$, etc. Rescaling $A^\mu \to eA^\mu$ we see both operators F^2 and $|D\Psi|^2$ possess canonical dimensions 2 and higher-derivative ones 4 and higher. Hence at long distances the latter is irrelevant in a RG point of view unless large anomalous dimensions are generated dynamically, the possibility of which cannot be excluded but shall not be considered here. Although we have so far not considered four and higher-point functions of (A, χ), it is straightforward to compute them. From the gauge invariance the results should anyhow consist of only higher powers of F and $D\Psi$ like $(F^2)^2$ or $(|D\Psi|^2)^2$ than $\mathcal{L}_{\text{Hig.}}$ and also of their higher derivatives. They are all irrelevant as seen from the naive dimensional counting. From the universality argument, $\mathcal{L}_{\text{Hig.}}$ should thus possess the same long-distance behavior as of $\mathcal{L}_{\text{inv.}}$.

Our derivation of the effective lagrangian is based on $1/N$ expansion and hence, from its validity, parameters β and κ defined by (2.18), should formally satisfy

$$\beta e^2 \gg 1, \qquad \kappa \gg 1. \tag{2.19}$$

From general interests in $\mathcal{L}_{\text{Hig.}}$ itself, however, we shall henceforth consider general regions of $\beta e^2 > 0$ and $\kappa > 0$. We expect that the qualitative properties at long distances are same between the full effective theory $\mathcal{L}_{\text{inv.}}$ and its lowest-derivative part $\mathcal{L}_{\text{Hig.}}$, for a general N satisfying $1 \ll N \leq \infty$.

To summarize this section we have constructed the long-distance effective lagrangian for the axially U(1)-gauged four-Fermi theory in $1+1$ dimensions, which reads

$$\mathcal{L}_{\text{eff.}} = \mathcal{L}_{\text{Hig.}} + \mathcal{L}_{\text{anom.}}, \quad \mathcal{L}_{\text{anom.}} = (\,b\,N/8\pi)A^2, \quad b \geq 0. \tag{2.20}$$

3 Long-distance properties of the frozen U(1) Higgs model and a massive s-G system

In this section we consider the effective theory in the gauge-invariant scheme ($b = 0$), i.e. the frozen U(1) Higgs model defined with the lagrangian $\mathcal{L}_{\text{Hig.}}$ in (2.17). As does so in $\mathcal{L}_{\text{inv.}}$, without singular configurations for a phase order-parameter χ the Higgs mechanism operates in this model; the theory is reduced to that of a

neutral massive vector boson with a n
U(1) charge is predicted to be screen
changed once we allow singular confi
are vortices in a euclidean space-time
cannot be absorbed into the vector fi
(2.15).

The many-body dynamics of vor
model (2.17) has been argued in sever
different approaches were mainly take
they relied upon a so called dilute-ins
ums [23, 24, 25]. The main results are
vacuums is proportional to $-\cos\theta$ an
where the global gauge symmetry is r
random varying of phase χ): the gaug
in addition to the massive pole at p^2 =
only if Q is an integral multiple of e
distance picture in this semi-classical
Higgs picture.

For these consequences to be justi
tant but unsolved problem *to verify dy*
relevant and their distribution is suff
this problem we must probe the long
mechanically, in other words, based o
been performed.

Another typical approach [26, 27]
transform it to a system of a spin-wave
dual transformation [29, 28]. In this ap
were proposed in a wide range of para
distance behavior requires again a prop
on a real space-time lattice and, to ou

In the rest of this section we will st
Higgs model (2.17) with the method b
We keep the system (2.17) in the contin

our long-distance effective lagrangian is valid, and take into account explicitly an ensemble of single-vortex configurations in the partition function. Then we shall proceed along the similar line of the lattice approach [26, 27]. Namely, we first transform the model to a scalar-vortex system by the continuum version of a dual transformation [30], and next generalize it by adding a chemical potential of vortices, controlled by a fugacity parameter y. The small-y regime ($0 < y \ll 1$) of this generalized system can be described by a local field theory, a massive s-G system (subsect.3.1). In this system we will be able to study the long-distance relevance of vortices in a systematic manner, i.e. by incorporating interaction effect among vortices through a momentum-shell recursion-relation method (subsect.3.2).

3.1 Dual transformation in the cut-off continuum and a s-G system

We start from the following euclidean partition function of the frozen U(1) Higgs model (2.17) with an ensemble of single-vortex configurations:

$$
\begin{aligned}
Z &= \prod_x \int \mathcal{D}A(x) \prod_{x_0} \sum_{n(x_0)} \prod_x \int \mathcal{D}\chi(x) \delta\left[\,\chi(x) - n(x_0)\,\theta(x-x_0)\,\right] \\
&\quad \exp\left\{ -\int d^2x \left[\frac{\beta}{4} F^2 + \frac{\kappa}{2} (A - \partial\chi)^2 \right] \right\}, \qquad (3.1)
\end{aligned}
$$

where x_0 are the positions of vortex centers and $\theta(x-x_0)$ denotes the azimuthal angle of x around x_0. In (3.1) the case $n=0$ corresponds to a usual unitary gauge ($\chi = 0$). Here we sum up over all vortex charges $n(x_0) \in Z$ defined at all possible centers x_0. The vortex centers are assumed to be separated from each other with the minimal length $a \equiv \Lambda^{-1}$ where Λ is the cut-off scale we are considering; in our effective theory of the four-Fermi model it is naturally given by the scale of order ρ_c. Performing the continuum version [30] of a dual transformation [29, 28]

$$
\exp\left\{ -\frac{\beta}{4} \int d^2x F^2 \right\} = \prod_x \int \mathcal{D}\phi \exp\left\{ -\int d^2x \left[\frac{1}{2\beta} \phi^2 - i\phi\,(\epsilon \cdot \partial A) \right] \right\} \qquad (3.2)
$$

with $\epsilon \cdot \partial A \equiv \epsilon_{\mu\nu}\partial_\mu A_\nu$ ($\epsilon_{12} = -\epsilon_{21} = 1, \epsilon_{11} = \epsilon_{22} = 0$), and integrating over the gauge field A, we obtain

$$
Z = \prod_x \int \mathcal{D}\phi(x) \prod_{x_0} \sum_{n(x_0)} \prod_x \int \mathcal{D}\chi(x) \delta\left[\,\chi(x) - n(x_0)\,\theta(x-x_0)\,\right]
$$

$$\exp\left\{ -\int d^2x \left[\frac{1}{2\kappa} (\partial\phi \right.\right.$$

where

$$p(x) \equiv ($$

is a vortex source. The value of $p(x)$

singular $\chi(x) = n(x_0)\theta(x-x_0)$ impos

$n(x_0)\,\delta^2(x-x_0)$. Then, integrating ov

$$
\begin{aligned}
Z &= \prod_x \int \mathcal{D}\phi(x) \prod_{x_0} \sum_{n(x_0)\in Z} \\
&\quad \exp\left\{ -\frac{1}{2\kappa} \int d^2x \left[(\partial\phi) \right.\right.
\end{aligned}
$$

which describes the system of many vo

ϕ. The integration over $\phi(x)$ provides

$$Z = Z^{\rm free}_{\rm sca.} \prod_{x_0} \sum_{n(x_0)\in Z} \exp\left\{ -2\pi^2\kappa \right.$$

where $Z^{\rm free}_{\rm sca.}$ represents the partition f

the presence of gauge interaction (β

short-ranged

$$D(x-y;M) =$$

in contrast to a long-range logarithmic

model. On the coincidence point $x =$

$$D(0;M) = (4$$

with $Z \equiv M^2\Lambda^{-2}$, which shows that a

$M^2 > 0 (\beta < \infty)$. Thus, if we neglect t

the partition function is written by

$$Z^{\rm free}_{\rm vor.} = \prod_{x_0} \sum_{n(x_0}$$

from which the probability $P(x_0)$ to fi

$$P = \Big($$

with $X \equiv \pi\kappa - 2$, and does the chemical potential as

$$\frac{\pi\kappa}{2} \ln\left(\frac{1+Z}{Z}\right). \tag{3.11}$$

Eq.(3.7) shows that the interaction length r among vortices is roughly $r \sim M^{-1} = Z^{-1/2}a$ and so the area in which a vortex can affect is estimated by $(r/a)^2 \sim Z^{-1}$ in the unit of a. Hence, from (3.10) the number of other vortices lying closely enough to interact with a given vortex reads approximately as

$$n^* \equiv Z^{\frac{X}{2}}(1+Z)^{-(1+\frac{X}{2})}. \tag{3.12}$$

We expect this expression to be qualitatively valid also for a smooth momentum cut-off. Although this expression of n^* is derived from a classical picture we can also define an effective one

$$n^*(\ell) = Z(\ell)^{\frac{X(\ell)}{2}}(1+Z(\ell))^{-(1+\frac{X(\ell)}{2})}, \tag{3.13}$$

for which quantum interaction effects are to be taken into account through a RG analysis. Here the dependences of $X(\ell)$ and $Z(\ell)$ on a scaling parameter ℓ are to be determined by RG. The quantity $n^*(\ell)$ thus changes dynamically as a function of ℓ and measures how vortices are effectively dilute (or dense) at long distances $(\ell \to \infty)$. $P(\ell)$ is defined similarly as well.

Now, we generalize the system (3.5) by introducing an additional chemical potential for a single vortex, the prescription having been developed for the study of the XY model [29]. This leads us to

$$\begin{aligned} Z(y) &= \prod_x \int \mathcal{D}\phi(x) \sum_{n(x_0)\in Z} \exp\Big\{-\frac{1}{2\kappa}\int d^2x \left[(\partial\phi)^2 + M^2\phi^2\right] \\ &\quad +(\ln y)\sum_{x_0} n(x_0)^2 + 2\pi i \sum_{x_0} n(x_0)\,\phi(x_0)\Big\}, \end{aligned} \tag{3.14}$$

where a chemical potential term $(\ln y)\, n^2(x_0)$ has been incorporated by hand to control fluctuations in $n(x_0)$. Although the original model corresponds to $y=1$, we will henceforth consider this generalized system $Z(y)$, regarding y as a free parameter (a fugacity) which controls the activation of single vortices. In the generalized system the probability P of a single-vortex excitation and the quantity n^* defined respectively in (3.10) and (3.12) are multiplied by y. Our interests

then lie in how this parameter y beha
especially in whether there exists the
long-distance limit $(\ell \to \infty)$. If such a
distance effect of vortices and the mea
is technically difficult to directly exar
of y. However, it is interesting and
to study how the system $Z(y)$ dynar
$(y \ll 1)$ of vortex activation [29].

In the region $y \ll 1$, the sum over

$$\ln \sum_{n(x_0)} \exp\Big\{(\ln y$$

$$= \ln\Big\{1 + 2\sum_{n(x_0)=1}^{\infty}$$

$$= \sum_{m\in N} y_m \cos\left[2\pi\right.$$

with

$$y_1 = 2y\,[1$$

$$y_2 = -y^2\,[$$

$$y_3 = (2/3)$$

$$y_4 = y^4 +$$

and $y_m (m \geq 5) \sim \mathrm{O}(y^5)$. In (3.16) onl
contributes to the most dominant term
in y_4 and higher-order harmonics, a d
the leading contributions. Rescaling ϕ
the continuum expression we get

$$Z_{\mathrm{msG}} = \prod_x \int \mathcal{D}\phi(x)\, \mathrm{ex}$$

$$-\Lambda^2 \sum_{m\in N} y_m$$

with y_m given by (3.16). As in the lat
massive s-G system starting from the

$y_m(\ell)$ as independent coupling constants and investigating their RG behaviors, we can perform the systematic study of the dynamical problem: whether vortices are relevant or not at long distances.

Before proceeding to the actual RG analyses let us give some remarks.

In ref. [27] Jones et al. have commented on the RG behavior of the massive s-G theory derived from the lattice frozen U(1) Higgs model. They argued in favor of the KT phase transition and predicted an existence of the phase ($\kappa < \kappa_c$) where the Higgs mechanism operates. However, their argument is based on a hypothesis that in each of the steps of RG the super-renormalizable mass term is irrelevant and can be ignored. This hypothesis may be plausible for the short-distance behavior but not so for the long-distance behavior which determines the phase structure of the model. Classically a massive term is infrared relevant. However, as is seen in the massless s-G theory (pure XY model) [32], a cosine operator which has the classical mass dimension zero, could acquire large (=2) anomalous dimensions from quantum corrections so that there can appear the (low-temperature) KT phase where vortices are irrelevant at long distances ($y_m(\ell \to \infty) \to 0$). Therefore, it is a non-trivial dynamical problem how the massive s-G system (3.17) behaves at long distances. To draw definite conclusions we must carefully investigate the mixed renormalization-effects among parameters κ, M^2 and y_m.

For this purpose it is important that RG equations should be constructed in terms of a recursion-relation method, as has been done for the massless s-G theory in a real space-time [32] and in a momentum space [33, 34]. Since from the beginning we are working in the continuum with an ultraviolet cut-off, it is natural to adopt the latter, i.e. a momentum-shell method. It should be stressed that other conventional field-theoretical schemes such as those based on *ultraviolet* divergences [35] are *not* suited for our purpose; in such schemes the long-distance effect of super-renormalizable terms such as a mass term could erroneously be missed. This fact is actually exemplified in the standard O(N) vector non-linear σ model with a *finite* external magnetic field h, defined with the lagrangian

$$\mathcal{L} = \frac{1}{2t}\left[(\partial\vec{\pi})^2 + \frac{(\vec{\pi}\cdot\partial\vec{\pi})^2}{1-\vec{\pi}^2}\right] + \frac{h}{t}(1-\vec{\pi}^2)^{1/2}, \tag{3.18}$$

where t is a coupling constant (temperature). In the conventional scheme of dimensional renormalizations [36] the last term is regarded only as an infrared cut-off

and does not affect the ultraviolet di
the result there appears no dependenc
is correct for the case with no exter
h in the critical region near the ultra
for the non-zero finite h was first der
momentum-shell recursion-relation me
behavior revealing a crossover phenon
field-theoretical approach relying on s
ence between these schemes also occu
σ model with no external fields [37].

We think that these failures are
could commonly happen in those utiliz
equations. Thus it seems essentially in
of the system (3.17) with the *recursio*
Wilson [31]. This approach of the mo
a massless s-G theory [33, 34] and it
system (3.17).

3.2 Construction of recurs

As has been argued above we sh
relation method to study correctly th
system (3.17). The applications of the
appeared in refs. [33, 34], and we mainl
the system should start from the smal
suppressed by a large chemical potent
can be treated by a perturbation the
order harmonics possess the coefficien
only cosine operators with coefficients
($m \geq 4$) harmonics may also be relev
at least initially, negligible and does n
of lower-order harmonics ($y_1 \sim y_3$). If
to zeroes, then the system in the long
boson with the mass $M^2 = \kappa/\beta$, in co

perturbation theory of the original U(1) Higgs model (2.17). Our main interest then lies in whether there could exist the parameter region in which such a Higgs picture is valid. This question can legitimately be examined by a small-fugacity ($|y_m| \ll 1$) perturbation theory.

The action of the massive s-G system consists of a free part S_0 of the massive scalar field and of its harmonic term S_I originated from the vortex configurations in the U(1) Higgs model, i.e. $S_{\mathrm{msG}} = S_0 + S_I$ where

$$S_0 = \frac{1}{2}\int d^2x \left[(\partial\phi)^2 + M^2\,\phi^2 \right], \qquad (3.19)$$

$$S_I = -\Lambda^2 \sum_{m=1} y_m \int d^2x \cos m\,\varphi_x \qquad (3.20)$$

with $\varphi_x \equiv 2\pi\sqrt{\kappa}\phi_x \equiv 2\pi\sqrt{\kappa}\phi(x)$. In the momentum-shell renormalization, the field ϕ is decomposed into a low ϕ' ($|p| < \Lambda' = \Lambda - d\Lambda$) and a high h ($\Lambda' < |p| < \Lambda$) momentum parts

$$\phi = \phi' + h, \qquad (3.21)$$

in which is assumed a smooth momentum-slicing procedure [31, 34]. Although this procedure is not explicitly specified here, it is in fact argued in ref. [31] that such a slicing procedure exists in principle. As is shown in refs. [33, 34] the present treatment applied to the massless s-G system in fact gives the same recursion equations as those obtained by other methods including those based on local singularities, at least to leading order [32, 35]. For the present *massive* s-G system defined at large distances we rather believe that the momentum-shell recursion relation method shall be more appropriate than others, by the reason we have remarked in subsect.3.1.

Owing to the above field decomposition the partition function of the system (3.17) reads

$$Z_{\mathrm{msG}} = \int \mathcal{D}\phi' \exp\{-S_0(\phi')\}\; Z' = \text{const.}\int \mathcal{D}\phi' \exp\{-F(\phi')\},$$

$$Z' = \int \mathcal{D}h\, \exp\{-S_h - S_I(\phi'+h)\}. \qquad (3.22)$$

Here $S_h \equiv S_0(h)$ is a free massive scalar-field action for the high-momentum part h and $F(\phi')$ represents a free energy of the low-momentum part ϕ'. Our task is to calculate $F(\phi')$ by a cumulant expansion with respect to S_h. Explicitly,

$$F(\phi') = S_0(\phi') - \ln(Z' Z_h^{-1})$$

$$= S_0(\phi') + < S_I > \ldots$$

$$+(1/6)\,(< S_I^3 > \ldots$$

$$+ \ldots,$$

where S_I stands for $S_I(\phi'+h)$ and th…

$$< S >= \int \mathcal{D}h\, S\, \exp(-S \ldots$$

In Appendix A we have computed o… the renormalizations of non-derivative… The influence from other derivative … expected to be very small. Combining… total free-energy density for the low-m…

$$\mathcal{F} = (1/2)\left[1 + 2c\pi^2\delta^2\Delta y_1^2 d\ell\right] \ldots$$

$$-\Lambda^2[\,y_1 - y_1\delta\Delta(\,1 - 8\pi\delta\Delta \ldots$$

$$-\Lambda^2[\,y_2 - \delta\Delta(\,4y_2 - \pi\delta\Delta \ldots$$

$$-\Lambda^2\left[\,y_3 - \delta\Delta\,(\,9y_3\, -\, 8\pi \ldots\right.$$

$$+\,\mathrm{O}(y^4, (d\ell)^2),$$

where $d\ell \equiv \Lambda^{-1}d\Lambda = a^{-1}da,\ \delta \equiv X + \ldots$ c is a constant number dependent on … renormalizations of the field and the pa… be rewritten in the original form

$$\mathcal{F} = \frac{1}{2}\,(\partial\tilde{\phi}')^2 + \frac{\tilde{M}^2}{2}\,\tilde{\phi}' \ldots$$

where the quantities with a tilde denot… paring (3.26) with (3.25) we can deriv… tions:

$$\frac{dX}{d\ell} = -\frac{1}{8}\,\delta^3\Delta Y_1^2,$$

$$\frac{dY_1}{d\ell} = \Big[\,2\, -\, \delta\,\Delta\,\Big(\,1 - \ldots$$

$$\frac{dY_2}{d\ell} = 2\,Y_2\, -\, \delta\,\Delta\,\Big(\,4\,Y_2 \ldots$$

$$\frac{dY_3}{d\ell} \;=\; 2\,Y_3 \,-\, \delta\,\Delta\left(9\,Y_3 \,-\, 2\,\delta\,\Delta\,Y_1\,Y_2 \,+\, \frac{1}{12}\,\delta^2\,\Delta^2\,Y_1^3\right), \tag{3.30}$$

$$\frac{dZ}{d\ell} \;=\; \left(2 \,-\, \frac{1}{8}\,\delta^2\,\Delta\,Y_1^2\right) Z, \tag{3.31}$$

where we have rescaled $4\pi\,c^{m/2}\,y_m = Y_m$.

3.3 Solutions of recursion equations

Having constructed the RG equations we now solve them numerically and investigate the behavior of RG flows. For simplicity we set $c = 1$ in the numerical calculations. (The choice of c does not alter the qualitative properties of RG flows.) In the followings we fix the initial conditions for Y_m by $Y_1(0) = 0.1$, which means from (3.16) to take $y \simeq 4.0 \times 10^{-3}$. Then, respecting other relations in (3.16) provides $y_2 \sim -y^2 \sim -1.6 \times 10^{-5}$ and $y_3 \sim (2/3)y^3 \sim 4.2 \times 10^{-8}$, and the initial conditions for Y_2 and Y_3 are fixed as

$$Y_2(0)\,(= 4\pi\,y_2) \;=\; -2 \times 10^{-4}, \quad Y_3(0)\,(= 4\pi\,y_3) \;=\; 5 \times 10^{-7}. \tag{3.32}$$

Let us first see the solutions for the massless ($Z = 0$) s-G system to compare with those for the massive ($Z > 0$) system. Physically this system is related to the pure (non-gauged) XY model or a two-dimensional Coulomb-gas system, and the lowest-order solution of RG with only a single harmonics (Y_1) is already known [32, 34]. Here we will show the RG flows including the effects of both higher-order corrections and higher harmonics. The RG flows projected on the (X, Y_1) plane are depicted in fig.5. As is seen in this figure, their qualitative behaviors are the same as those obtained by the lowest-order ($\mathrm{O}(y_1^2)$) analyses [32, 34]. The flow diagram exhibits the well known two-phase structure distinguished by the KT phase boundary. As is read from (3.27)$\sim$ (3.30), the flow equations are approximated by

$$\frac{dX}{d\ell} \simeq -Y_1^2, \qquad \frac{dY_1}{d\ell} \simeq -XY_1, \qquad \frac{dY_2}{d\ell} \simeq \frac{dY_3}{d\ell} \simeq 0, \tag{3.33}$$

in the neighborhood of $X \simeq Y_m \simeq 0$ where the phase boundary in the (X, Y_1) plane is described by $X \simeq Y_1$. Figs.6(a) and (b) show the behaviors of $Y_1(\ell), Y_2(\ell)$ and $Y_3(\ell)$ as functions of a RG step ℓ, starting from the representative points in the small ($X(0) = 0.075$) and the large ($X(0) = 0.125$) X regimes. It is

observed that the higher harmonics
harmonic ($m = 1$), except in the first
(fig.6(a)), $Y_2(\ell)$ immediately ($\ell \ll 1$) c
decreasing at $\ell \sim 0.62$. Also $Y_3(\ell)$ initi
at $\ell \sim 0.74$. All $Y_{m(=1\sim 3)}(\ell)$ continue
$\ell = 13.25 \sim 13.75$ where they turn to
(fig.6(b)) all flows finally converge to t
although $Y_2(\ell)$ and $Y_3(\ell)$ behave agair

Since a single-charged ($n(x_0) = \pm 1$
bution to the second and the third har
behave similarly at long distances prov
namics. That is, the physical interpret
persists to hold in our extended system
XY model) regime constitutes the KT
chemical potential (small fugacities $|y$
tances. The long-distance physics is d
is characterized by a power-law decay
The non-gauged chiral four-Fermi mo
phase. In the small-X (high-temper
with a small chemical potential (large
correlation of the massless scalar field
the disordered (vortex) phase is realiz

How are the above features of the r
term is added ($Z > 0$), namely if the X
The recursion equations (3.27)$\sim$(3.30
fixed line of RG, irrespective of the val
is whether there still exists the region
line where single vortices are complete
is described by a free massive scalar fi

Fig.7 exhibits the behaviors of Y
$Z(0) = 0$, 0.01 and 0.1 with $X(0) = 1$
in the case of the massless ($Z = 0$) sy
phase ($X \gg Y_1$). It is shown for the r

$Y_1(\ell)$ stops at a certain $\ell(=\ell_c)$ the value of which depends on an initial condition and that $Y_1(\ell)$ turns to an exponential increasing at large $\ell(>\ell_c)$. Fig.8(a) shows the behaviors of $Y_1(\ell)$ and $Z(\ell)$ as functions of a step ℓ, where $Z(0)$ is chosen to be 0.01. $Z(\ell)$ remains small within small steps ($\ell < \ell_c \approx 2$), where the second term (anomalous dimensions of $y_1\Lambda^2$) in the square bracket on the right-hand side (r.h.s.) of the recursion equation (3.28) is larger than or comparable with the first term ($=2$, canonical dimensions of $y_1\Lambda^2$) and hence $Y_1(\ell)$ decreases obeying the *quantum* scaling as that for $Z=0$. However, after some steps ($\ell \sim \ell_c$) where $Z(\ell)$ reaches to be O(1), $Z(\ell)$ grows large and reduces the anomalous dimensions of $y_1\Lambda^2$ significantly, $\Delta(\ell \gg \ell_c) \approx 0$. As a result $Y_1(\ell)$ turns to an exponential increasing according to a *classical* scaling law (see also fig.8(b) plotted by a logarithmic scale). Fig.8(b) also indicates that this crossover and the scaling behaviors in the massive system are common in three harmonics $Y_m(\ell)_{(m=1\sim3)}$, although in this figure is comprised large-fugacity ($Y_m(\ell) \gg 1$) behaviors extrapolated from the perturbative results. We have checked numerically that these qualitative properties, more or less, hold for all initial conditions set in a large X regime. There are no flows converging in the long-distance limit to a $Y_m = 0_{(m=1\sim3)}$ fixed line. In any case including a small X regime flows eventually ($\ell \to \infty$) go to the region where all Y_m are large.

For the massive s-G system, we thus conclude that there is no phase transition nor a Higgs phase defined by $Y_m(\ell \to \infty) \to 0$. Instead we have observed crossover phenomena from the classical to the quantum scaling regimes in some intermediate scales (ℓ_c) which themselves depend on initial conditions. These crossover phenomena are driven by an increasing of the effective mass parameter $Z(\ell)$($\approx$ gauge coupling).

Fig.8(a) also tells us that the effective probability of a vortex in the generalized system defined by $yP(\ell) \approx (1/2)y_1(\ell)P(\ell)$ with P in (3.10) increases monotonically and that the quantity $y\,n^*(\ell) \approx (1/2)y_1(\ell)n^*(\ell)$ which roughly measures the effective vortex density in the generalized system remains almost constant (increases slightly), starting from the very dilute region 3.92×10^{-4}. Therefore it is not obvious whether the diluteness of the vortex distribution can actually be realized or not in the long-distance limit ($\ell \to \infty$) even if it is chosen so initially ($\ell = 0$) at some finite scales.

4 Long-distance prope
frozen U(1) Higgs

In this section we turn our attenti
(anomalous) schemes ($b \neq 0$). The effe
gauge-field mass term proportional to
model. We proceed again along the du

4.1 Double dual transform

As in the previous case, performing
ing over the gauge field we obtain the
$\mathcal{L}_{\text{Hig.}} + \mathcal{L}_{\text{anom.}}$:

$$Z = \prod_x \int \mathcal{D}\phi_1(x) \prod_x \int \mathcal{D}\chi(x)$$
$$\exp\left\{-\int d^2x \left[\frac{1}{2\kappa_+}(\partial\phi_1)\right.\right.$$

with

$$\kappa_+ \equiv \kappa + \frac{b\,N}{4\pi},$$

This system may be regarded as the
of (4.1)) coupled to a massive scalar
$(2\pi)^{-1}\epsilon_{\mu\nu}\partial_\mu\partial_\nu\chi(x)$.

The XY model lagrangian can be
troduce a fictitious gauge field A' and
performing the dual transformation (3

$$\exp\left\{-\frac{\kappa_0}{2}\int d^2x(\partial\chi)^2\right\}$$
$$= \prod_x \int \mathcal{D}A'(x) \exp\left\{-\int d^2x\right.$$
$$= \prod_x \int \mathcal{D}A'(x)\mathcal{D}\phi_0(x)$$
$$\exp\left\{-\int d^2x \left[\frac{1}{2\beta'}\phi_0^2 - i\right.\right.$$
$$= \prod_x \int \mathcal{D}\phi_0(x) \exp\left\{-\int d^2x\right.$$

where $\beta' \equiv (e')^{-2}$ and we have neglected overall constant factors. In the second equality of (4.3), we have rescaled $e'A' \to A'$ and introduced a new scalar field $\phi_0(x)$ via the dual transformation. After the double dual transformation (4.1) and (4.3), the χ dependence in the effective action emerges only through the vortex source operator $p(x)$. Hence the χ configurations which can affect the partition function, are only vortex configurations $\chi(x) = n(x_0)\,\theta(x-x_0)$ with arbitrary integer charges $n(x_0)$ and centers x_0, as has been specified in (3.1) by the delta functional as the gauge-fixing procedure for the gauge-invariant system.

Substituting (4.3) into (4.1) and integrating over χ with the above consideration, we get the following total partition function:

$$\begin{aligned} Z &= \prod_x \int \mathcal{D}\phi_0(x)\mathcal{D}\phi_1(x) \sum_n \exp\left\{ -\int d^2x \left[\frac{1}{2\kappa_0}(\partial\phi_0)^2 + \frac{1}{2\kappa_+}\Big((\partial\phi_1)^2 \right.\right. \\ &\quad \left.\left. + M_1^2\,\phi_1^2 \Big) \right] + 2\pi i \sum_{x_0} n(x_0) \left[\phi_0 + \left(\frac{\kappa}{\kappa_+}\right)\phi_1 \right](x_0) \right\} \end{aligned} \tag{4.4}$$

with $M_1^2 \equiv \kappa_+/\beta$. We thus have seen that our effective theory $\mathcal{L}_{\text{Hig.}} + \mathcal{L}_{\text{anom.}}$ is equivalent to a system of *vortices interacting with both massless (ϕ_0) and massive (ϕ_1) scalar fields*.

As in the previous case, in order to systematically study whether vortex excitations are actually relevant or not at long distances, we add a chemical potential term $(\ln y)\sum_{x_0} n^2(x_0)$ to reduce the vortex activation ($y \ll 1$). Then taking the sum over vortex charges and identifying the sum over vortex centers $\sum_{x_0}$ with the continuum integral $a^{-2}\int d^2x$, we obtain the following *coupled system of massless and massive s-G fields*:

$$\begin{aligned} Z &= \prod_x \int \mathcal{D}\phi_0(x)\mathcal{D}\phi_1(x) \exp\left\{ -\int d^2x \left[\frac{1}{2}(\partial\phi_0)^2 + \frac{1}{2}\Big((\partial\phi_1)^2 + M_1^2\,\phi_1^2 \Big) \right.\right. \\ &\quad \left.\left. -\Lambda^2 \sum_{m\in N} y_m \cos\left(2m\pi \sum_{j=0}^{1} \sqrt{\kappa_j}\phi_j \right) \right] \right\}, \end{aligned} \tag{4.5}$$

where

$$\kappa_1 \equiv \kappa^2/\kappa_+ = \kappa - \kappa_0, \tag{4.6}$$

and the fields have been rescaled as $\phi_0 \to \sqrt{\kappa_0}\phi_0$ and $\phi_1 \to \sqrt{\kappa_+}\phi_1$. Here $y_m = y_m(y)$ is of order $\mathrm{O}(y^m)$. Eqs.(4.1),(4.2) and (4.5) indicate that for the regularization scheme $b < 0$ ($\kappa_0 < 0$) the theory is non-unitary and we consider only the case $\kappa_0 > 0$ which is realized by the $b > 0$ schemes.

4.2 Recursion equations fo

Using again the recursion-relation me
erties of the coupled s-G system (4.5)
the original system (4.4) corresponds t
small $y(\ll 1)$ regime where the pertu
analysis persists us to speculate on wh
possess the phase where vortices are i
although we cannot have direct acces
previous system, vortices in the curre
through a short-ranged interaction du
due to the *massless* field ϕ_0. We thus
the coupled system (4.5) should be diff
massive system, even *qualitatively*.

The procedure of constructing the
is the same as that for the pure system
dynamical fields. They are decompose

$$\phi_j = \phi'_j + $$

the latter of which is integrated over
cursion relations. In the current anal
the $\mathrm{O}(y^2, \partial^2)$ approximation. As has b
unlikely that higher-order corrections
behavior in the $y \ll 1$ regime.

Now, in the initial point ($\ell = 0$) c
standard s-G lagrangian (4.5) with tw
tending the procedures developed in A
we calculate the renormalized free ene
with respect to the free part S_0 of the
is to be replaced by

$$< S >= \prod_{j=0}^{1} \int \mathcal{D}h_j\, S\, \mathrm{e}^{-}$$

with $S_h \equiv S_0(h_0, h_1)$ being a free kine

The effect of the first-order cumulant is merely to renormalize the fugacity parameters y_m of the original harmonics,

$$< S_I >= -\Lambda^2 \sum_{m=1}^{2} y_m \left(1 - m^2 \,\delta\, d\ell\right) \int d^2x \cos m\,\varphi' + \mathrm{O}(\,(d\ell)^2\,) \tag{4.9}$$

with $\delta \equiv \sum_{j=0}^{1} \Delta_j \delta_j,\ \Delta_j \equiv (1+Z_j)^{-1},\ Z_j \equiv \delta_{1j} M_1^2 \Lambda^{-2},\ \delta_j \equiv X_j + 2 \equiv \pi\kappa_j$ and $\varphi' \equiv 2\pi \sum_{j=0}^{1} \sqrt{\kappa_j}\phi_j \equiv \sum_{j=0}^{1} \varphi_j$. However, the second-order cumulant generates new operators as well as the wave-function renormalizations. Up to irrelevant higher-derivative operators, it reads

$$\begin{aligned}
& (-1/2)\left(< S_I^2 > - < S_I >^2\right) \\
= \; & \int d^2x \left\{ c\,\pi^2 \delta y_1^2\, d\ell \sum_{j=0}^{1} \delta_j\,(\partial\phi_j')^2 - \pi\, y_1^2 \delta^2 \Lambda^2\, d\ell\, \cos 2\varphi' \right. \\
& - (c\,\pi^2\,/\,8)\, y_1^2\, \delta\, d\ell \sum_{j=0}^{1} \partial^2 \varphi_j' \sin 2\varphi' \\
& \left. + 2\,c\,\pi^2 \sqrt{\delta_0\,\delta_1}\,\delta y_1^2\, d\ell\, \partial\phi_0' \partial\phi_1' + \mathrm{O}((d\ell)^2, \partial^4) \right\}.
\end{aligned} \tag{4.10}$$

Therefore, in order to complete the $\mathrm{O}(y^2, \partial^2)$ RG, we must start from the generalized action $S = S_0 + S_I'$ with the bare interaction

$$S_I' = \int d^2x \left[-\Lambda^2 \sum_{m=1}^{2} y_m \cos m\varphi - \frac{1}{2}\sum_{j=0}^{1} w_j \partial^2 \varphi_j \sin 2\varphi + v\, \partial\phi_0 \partial\phi_1 \right], \tag{4.11}$$

where v and w_j are new coupling constants. They are initially ($\ell = 0$) zeroes but are generated at $\ell > 0$ by the $\mathrm{O}(y^2)$ perturbation, as is manifested in (4.10). The second term in (4.11) produces new contributions to the first-order cumulant

$$\begin{aligned}
< S_I' > \; = \; & \int d^2x \left\{ -y_1 \Lambda^2 \,(1 - \delta d\ell) \cos\varphi' \right. \\
& -\Lambda^2 \left[y_2 - \left(4 y_2 \delta + \pi^{-1} W \cdot \delta\right) d\ell \right] \cos 2\varphi' \\
& - \frac{1}{2}\,(1 - 4\delta d\ell) \sum_{j=0}^{1} w_j \partial^2 \varphi_j' \sin 2\varphi' + v \partial\phi_0' \partial\phi_1' \\
& \left. + \mathrm{O}((d\ell)^2) \right\}
\end{aligned} \tag{4.12}$$

with $W \cdot \delta \equiv \sum_{j=0}^{1} W_j \delta_j \Delta_j$ and $W_j \equiv 4\pi w_j$. In the $\mathrm{O}(y^2)$ approximation, the second-order cumulant for the new action (4.11) is precisely the one in (4.10).

The sum of (4.10) and (4.12) is cas

density in the $\mathrm{O}(y^2, \partial^2\,)$ approximatio

$$\begin{aligned}
\mathcal{F}(\phi_0', \phi_1') \; = \; & (1/2) \sum_{j=0}^{1} \Big(1 + \\
& - y_1 \Lambda^2\,(1 - \delta d \\
& - \Lambda^2 \Big[\, y_2 - \Big(4 \\
& - \frac{1}{2} \sum_{j=0}^{1} \Big[\, w_j - \\
& + \Big(v + 2c\pi^2\delta\sqrt{}
\end{aligned}$$

Under the same approximation fol

$$\begin{aligned}
\frac{dX_j}{d\ell} &= -\frac{1}{8}\,\delta\,\delta_j^2\, Y_1^2, \qquad \frac{dY}{d\ell} \\
\frac{dY_2}{d\ell} &= 2\,Y_2 - \Big(4\delta Y_2 + 4c \\
\frac{dZ_j}{d\ell} &= \Big(2 - \frac{1}{8}\delta\delta_j Y_1^2\Big)\, Z_j, \\
\frac{dV}{d\ell} &= -\frac{1}{16}\Big(\sqrt{\delta_1} - \sqrt{\delta_2}\Big)
\end{aligned}$$

where the coupling constants are resc

essary, we can add the $\mathrm{O}(y^3)$ correctio

complicated. As the relatively feasible

to the renormalization of the $\cos\varphi$ ope

replacements in the r.h.s. of the secon

$$(\,2 - \delta\,)\, Y_1 \longrightarrow \Big[\, 2$$

the effect of which is negligible in the

4.3 Solutions of recursion system

Before presenting the full numerica

the analytical behavior of (4.14) at sor

the relations (2.18), (4.2) and (4.6) lea

$$\kappa_0 \approx \frac{N}{4\pi}\,\frac{b}{1+b}$$

and for the special choice $b = 1$

$$(X \equiv) X_0 = X_1 = \frac{N}{8} - 2, \tag{4.20}$$

on which initial conditions for X_0 and X_1 are identical and the equality is not altered by the RG (4.14), that is $X_0(\ell) = X_1(\ell)$ for any step ℓ.

First, in the strong gauge-coupling limit $Z_1 \to \infty$ recursion equations in (4.14) are equal to those for the pure massless s-G system in the second-order approximation ((3.27) and (3.28) with $Y_2 = Z = 0$ and $Y_1^3 \to 0$). They describe the KT transition with the phase boundary $X_0 = Y_1$ (fig.5).

Next, if $Z_1 = 0$ [3] the system is a two-component massless s-G system in which recursion equations in (4.14) are reduced to

$$\frac{dX_j}{d\ell} = -\frac{1}{8} X_j^2 \,(X_0 + X_1 + 4) Y^2, \qquad \frac{dY_1}{d\ell} = -\,(X_0 + X_1 + 2) Y_1. \tag{4.21}$$

The critical line exists at $Y_1 = 0$, $X_0 + X_1 + 2 = 0$ which corresponds to $N = 24$ in the large-N approximation (4.19) with an arbitrary $b > 0$. Fig.9 exhibits the RG flows ($b = 1$) projected on a (X, Y_1) plane. The phase boundary around the critical point $(X = -1, Y_1 = 0)$ is written by

$$x^2 - \frac{y^2}{8} = 0 \tag{4.22}$$

with x, y denoting small deviations from the critical point, i.e. $X = -1 + x, Y_1 = y$.

Roughly speaking, if the mass parameter $Z_1(\ell)$ of ϕ_1 would scale up at long distances ($\ell \to \infty$) as for the pure massive s-G system (sect.3), the RG flows, starting from a very small but a non-zero $Z_1(0)$, should exhibit a crossover from the two-component massless s-G ($Z_1 = 0$) to the pure massless s-G ($Z_1 \to \infty$) systems. The critical behavior in the long-distance limit shall thus be described by the latter.

Fig.10 reveals the RG flows numerically solved for the coupled s-G system with $b = 1$. They are projected on the (X, Y_1) plane and the initial value of Z_1 is taken to be $Z_1(0) = 0.1$. We have checked that $Z_1(\ell)$ in fact grows large at long distances. Accordingly, the crossover behavior can be seen as expected above. Since $Z_1(0)$

[3] Do not confuse this case with the non-gauged ($e = 0$) limit of the original four-Fermi model, where there is no anomaly from the beginning. The non-gauged model ($\approx$ XY model) is related to the one-component massless s-G system (sect.3).

is small the flows are initially obeyed
($Z_1 = 0$). Beyond some steps where Z
ϕ_1 starts to be frozen and the flows a
massless s-G system ($Z_1 \to \infty$). As
the critical point $X_0 = Y_m = 0$, and
regime. In the zero-th order approxim

$$b\,\pi\,\kappa\,N - 8$$

which reduces in the large N limit to

$$N >$$

Note that this critical point $X_0 = 0$ d
choice of a slicing procedure, as is the
is seen in fig.10, since the phase bound
the Y_1 axis than that for a pure massl
(due to non-zero Y_1) to the above est
those for the latter.

In the KT phase, vortices are irr
in the long-distance theory consists o
massive scalar field (ϕ_1) with the mas

$$\frac{M_1^2}{1 - V}$$

which scales up at long distances. In
($N_c \to \infty$). This consequence is consis
s-G system (sect.3) deduced from the

The similar critical behaviors are
w_j. Figs.11(a) and (b) show the RG
and $10^2 W$ as functions of ℓ, in the vort
figures we have chosen $b = 1$ so that V
vortex phase (fig.11(a)) all the coupli
whereas they are irrelevant in the KT

[4] Although we plotted in fig.11(a) only th
to converge to zero values at sufficiently lon

5 Summary and Concluding remarks

Previous studies of the so called dynamical symmetry breaking in the (gauged) four-Fermi models has mostly relied upon the mean-field type treatments. As a simple and typical case, the naive application of $1/N$ expansion to the axially U(1)-gauged four-Fermi theory in $1+1$ dimensions predicts the dynamical Higgs phenomenon [7] which, in contrast to the non-gauged model [8], is stable at least qualitatively against higher-order corrections. In this article we have then addressed the problem whether such a picture is really valid or not, and have investigated the possibility that topological excitations (vortices) of the model could be dynamically relevant to the long-distance properties. The latter issue is important for the former general question because topological excitations are non-mean-field-like objects and, if relevant, usually work toward disordering the system and restoring the symmetry broken by the mean-field ansatz, as is known in the dilute-instanton-gas arguments. This problem is of general importance, with no special regard to the space-time dimensionality.

To simplify the problem we have first derived the large-N long-distance effective lagrangian along with the Witten's prescription in a non-gauged model [10]. We have kept a global chiral-U(1) symmetry without assuming any v.e.v. for the angle part χ of the axial-U(1) order-parameter $\sigma+i\pi$, and have put a v.e.v. for the radial part the value of which is determined by the large-N gap equation. The derived two-point effective action consists of the non-local part which is (axially) U(1) gauge-invariant, and of the contact mass term for the gauge field the coefficient (b) of which is arbitrary and depends on a fermion-loop regularization scheme.

If the (axially) gauge-invariant scheme is chosen, one only obtains the non-local part. This part displays the dynamical Higgs mechanism in a gauge-invariant fashion *provided* χ is restricted to be single-valued. The lowest-derivative part of this non-local action is a frozen U(1) Higgs model with two parameters. For this effective theory we have studied the long-distance relevance of multi-valued (vortex) configurations of χ. With the help of the dual transformation, the partition function of the frozen U(1) Higgs model with an ensemble of single-vortex configurations has been shown to be equivalent with the system of many vortices interacting with a massive scalar field. This system has been further generalized

to incorporate an additional chemical-
a new parameter y which controls th
we have examined the dynamical respo
tex activation. The small y regime of
system with small fugacities ($y_m \sim$ O(
tive recursion-relation method in a m
(Higgs) phase characterized by $y_m(\ell$ -
KT phase in the massless s-G system
hence *vortices are always relevant at*
crossover scale ($\ell = \ell_c$), we have obs
due to a rapid increasing of the effecti
effective fugacities $y_m(\ell)$ obeys a class

Therefore it is strongly suggested
effective theory of the axially U(1)-gau
scheme, corresponding to the $y = 1$ l
does *not* reach in the long distance lin
completely irrelevant and the Higgs me
due to a general disordering (symmet
we shall *not* be allowed to expect th
macroscopically.

If the (axially) gauge non-invaria
fermion loops, the effective theory of
U(1) Higgs model with a non-zero gau
we have also studied the long-distance
Utilizing the double dual transformat
alent to the system of vortices interac
fields. This system has been mapped
system of massless and massive s-G fie
performed with the result that a KT-t
of the Higgs coupling which is given t
in the small y regime (fig.12(b)). Th
given in Appendix B suggest that in t

present four-Fermi theory undergoes a KT-type phase transition at

$$N = N_{\mathrm{c}} \approx \frac{8(1+b)}{b}, \tag{5.1}$$

in the number N of fermion species (fig.12(b)). The large-N ($> N_{\mathrm{c}}$) regime of the system constitutes a KT-like phase where topological excitations are irrelevant and the long-distance properties are characterized by a free massless scalar field. This phase disappears ($N_{\mathrm{c}} \to \infty$) in the gauge-invariant limit ($b \to 0$) at which κ_0 is zero and the massless field is frozen. In this limit the theory is thus consistently continued to the gauge-invariant scheme ($b = 0$), and the combined results in both gauge-invariant and non-invariant schemes provides a unified view for the phase structure of the four-Fermi model in general schemes $b \geq 0$.

There are some points left to be clarified such as to determine the macroscopic order parameter characterizing the phase structure. To make the phase structure more precise and concrete it would be necessary to consider the v.e.v. of a Wilson loop operator [27] without relying upon a dilute-instanton-gas approximation. One unanswered technical problem in our recursion-relation analysis is that we have assumed a smooth momentum slicing procedure but have not specified it explicitly. Although the general possibility of taking such a slicing is argued in literatures [31] it would be desirable to present it in the explicit form.

In refs. [38], Banks et al. and Halpern studied the bosonization of the non-gauged SU(N) Thirring model. It may be an interesting further direction to apply the method to the present gauged model and to compare the results with those obtained here.

Anyhow the analysis developed in this article presents the prototype that the long-distance picture of the dynamical symmetry breaking based on a naive mean-field treatment should be modified even *qualitatively* due to the *topological* effects. Note that *their existence itself is not special to* $1+1$ *dimensions nor to the abelian nature of the gauge group*. Then, it may be suspected that similar modifications of the long-distance picture *could* occur as well in higher-dimensional four-Fermi and Higgs models icluding those with non-abelian gauge symmetries. A semi-classical argument supporting this conjecture already exists in a certain model [39, 28]. In the RG point of view one important difference between four and lower than four dimensions may be that the gauge coupling constant is dimensionless in the former

while has positive mass dimensions in
in the gauge non-invariant scheme, th
the long-ranged logarithmic interactio
the long-distance properties of the sy
logical procedures developed here wil
the long-distance *quantum* dynamics i

Our results obtained for the gauge
some low-dimensional systems in solic
ductivity mentioned in the introductio
Meissner effect would not occur at finit
is to be treated *quantum* mechanically
tions. Although the KT-type phase tra
scheme is very interesting, we are no
mass term of the gauge field could b
systems.

With the different motivation from
performed a recursion-relation analys
harmonic ($m = 1$) and drew the simila
U(1) Higgs model.

The author would like to thank I. I
K. Shizuya for useful discussions and c
members of YITP (Uji) for their helps

Appendix A. Evalua recursion-relation analy

In this appendix we present the
recursion-relation analysis of the mass
the original kinetic term $(\partial\phi)^2$ we con
erators $\cos m\varphi$ within $\mathrm{O}(y^3)$ approxim
the procedure to that applicable for th
where instead we take into account al

The first-order cumulant $< S_I >$ is calculated easily with the result

$$< S_I >= -\Lambda^2 \sum_{m=1} y_m A^{m^2}(0) \int d^2x \cos m\varphi'_x, \tag{A.1}$$

where $A(x) = \mathrm{e}^{-G_x}$ is defined with the Green function G_x for h

$$(2\pi^2\kappa)^{-1} G_x =< h(x)h(0) >= \int_{\Lambda'<|p|<\Lambda} d^2p \, \frac{\mathrm{e}^{ip\cdot x}}{p^2 + M^2}. \tag{A.2}$$

To $\mathrm{O}(d\ell = \Lambda^{-1} d\Lambda)$ we evaluate

$$A^{m^2}(0) = 1 - m^2 \pi\kappa (1+Z)^{-1} d\ell. \tag{A.3}$$

In the second-order cumulant we consider terms of $\mathrm{O}(y_1^2) \sim \mathrm{O}(y^2)$ and of $\mathrm{O}(y_1y_2) \sim \mathrm{O}(y^3)$

$$\begin{aligned}
& (-1/2)(< S_I^2 > - < S_I >^2) \\
= \;& (-1/4) y_1^2 A^2(0) \Lambda^4 \int d^2x \int d^2y \\
& [\,(A_{xy}^2 - 1)\cos(\varphi'_x + \varphi'_y) + (A_{xy}^{-2} - 1)\cos(\varphi'_x - \varphi'_y)\,] \\
& +(1/2) y_1 y_2 A^5(0) \Lambda^4 \int d^2x \int d^2y \\
& [\,(A_{xy}^4 - 1)\cos(\varphi'_x + 2\varphi'_y) + (A_{xy}^{-4} - 1)\cos(\varphi'_x - 2\varphi'_y)\,] \\
\simeq \;& (1/4) y_1 \pi^2 \kappa A^2(0) \Lambda^2 \int d^2\xi \xi^2 \, C_\xi \int d^2z (\partial\phi'_z)^2 \\
& +(1/2) y_1 y_2 A^5(0) \Lambda^4 \int d^2\xi \int d^2z [\, C_\xi(2 + C_\xi)\cos\varphi'_z + B_\xi(2 + B_\xi)\cos 3\varphi'_z\,] \\
& -(1/4) y_1^2 A^2(0) \Lambda^4 \int d^2\xi \, B_\xi \int d^2z \cos 2\varphi'_z,
\end{aligned} \tag{A.4}$$

where $\xi \equiv x - y$, $z \equiv (x+y)/2$, $A_{xy} \equiv A(x-y)$, $B_\xi \equiv A^2(\xi) - 1$, $C_\xi \equiv A^{-2}(\xi) - 1$. In the present approximation taken in subsect.3.2 we have neglected in the last semi-equality higher-derivative terms such as $(\partial\varphi')^{2n}_{(n\geq 2)}$, $(\partial\varphi')^{2n} \cos m\varphi'_{(n\geq 1)}$ and $(\partial\varphi')^{2n} \sin m\varphi'_{(n\geq 1)}$. To $\mathrm{O}(d\ell)$ we evaluate

$$\begin{aligned}
\int d^2\xi \, B_\xi &= \int d^2\xi \, C_\xi = \frac{1}{2}\int d^2\xi \, B_\xi^2 = \frac{1}{2}\int d^2\xi \, C_\xi^2, \\
&= 4\pi^3\kappa^2 (1+Z)^{-2} \Lambda^{-3} d\Lambda,
\end{aligned} \tag{A.5}$$

$$\int d^2\xi \xi^2 \, C_\xi = 4\, c\, \pi^2 \kappa (1+Z)^{-1} \Lambda^{-5} d\Lambda, \tag{A.6}$$

where $c \equiv \int d\lambda \lambda^3 \tilde{J}_0(\lambda)$ is a dimensionless constant depending on a momentum-slicing procedure [34, 31]. Here $\tilde{J}_0(\lambda)$ denotes a Bessel function modified so that λ integral converges [34] according to

not specified here. Inserting (A.5) and

$$\begin{aligned}
& (-1/2)(< S_I^2 > - \\
\longrightarrow \;& c\,\pi^4\kappa^2 y_1^2 (1+Z)^{-1} \\
& + 8\pi^3\kappa^2 y_1 y_2 (1 + Z \\
& - \pi^3 \kappa^2 y_1^2 (1+Z)^{-}
\end{aligned}$$

Similarly we evaluate the third-ord

$$\begin{aligned}
& (1/6)(< S_I^3 > -3 < S_I^2 \\
= \;& \frac{1}{24} A^3(0) y_1^3 \Lambda^6 \iiint d^2x \\
& [\,(A_{xy}^2 A_{yz}^2 A_{zx}^2 - A_{xy}^2 - \\
& (A_{xy}^2 A_{yz}^{-2} A_{zx}^{-2} - A_{xy}^2 - \\
& + (\text{cyclic permutations} \\
\simeq \;& \frac{1}{24} A^3(0) y_1^3 \Lambda^6 \iiint d^2\xi \\
& [\,(B_\xi B_\eta + B_\eta B_{\xi+\eta} + B \\
& + 3\,(B_\xi C_\eta + C_\eta C_{\xi+\eta} +
\end{aligned}$$

To $\mathrm{O}(d\ell)$, a careful evaluation of ξ, η i

$$\begin{aligned}
& (1/6)(< S_I^3 > -3 < S_I^2 \\
\longrightarrow \;& -\frac{1}{3} A^3(0) y_1^3 \Lambda^6 \iint d^2\xi \, d^2 \\
\longrightarrow \;& -4\pi^5\kappa^3 y_1^3 (1+Z)^{-3} \Lambda^2 d
\end{aligned}$$

Appendix B. An appr
$b > 0$ system

We have seen in sect.4 that the c at least in the region $X_0 > 0$ near Y only implies that our effective lagrang since the original system (4.4) corres perturbation is not valid. Since the coupled s-G system exist for *any* non-zero value of X_0 and since we have considered operators being most infrared-important in a RG sense, it is plausible that our original model with a sufficiently large N is indeed in the KT phase. Although it is difficult to give the proof, we will develop here an approximate argument supporting partly this conjecture. The idea is as follows. Starting from the system (4.4), we integrate over all massive (ϕ_1) scalar modes ($0 < |p| < \Lambda$). In a certain region of parameters e and N, the short-ranged interaction among vortices due to ϕ_1 can be neglected. The system is then well approximated by that only of a massless scalar field (ϕ_0) coupled to vortices with a sufficiently large chemical potential which is generated by the ϕ_1 fluctuation. Without introducing any additional chemeical potential the system is now described by the massless s-G system with a small fugacity $y \ll 1$, for which the perturbative RG analysis directly applies.

First, the integration over all scalar fields ϕ_0 and ϕ_1 in (4.4) leads to the effective action

$$2\pi^2 \sum_{x_0,y_0} n(x_0)\,[\,\kappa_0 D(x_0-y_0;0) + \kappa_1 D(x_0-y_0;M_1)\,]\; n(y_0). \tag{B.1}$$

As in the pure XY model, an infrared divergence in $D(0;0)$ requires a neutrality condition $\sum_{x_0} n(x_0) = 0$. Under this condition the effective action (B.1) can be rewritten as

$$\begin{aligned} &\left[2c_0\pi^2\kappa_0 + \frac{\pi\kappa_1}{2}\ln\left(\frac{1+Z_1}{Z_1}\right)\right]\sum_{x_0} n^2(x_0) \\ +\;\; &2\pi^2 \sum_{x_0\neq y_0} n(x_0)\,[\,\kappa_0 D'(x_0-y_0;0) + \kappa_1 D(x_0-y_0;M_1)\,]\; n(y_0), \end{aligned} \tag{B.2}$$

where c_0 is a finite positive constant and D' is defined by

$$\begin{aligned} D'(x;0) &\equiv D(x;0) - D(0;0) - c_0 \\ &\xrightarrow{|x|\Lambda\to\infty} -\frac{1}{2\pi}\ln|x|\Lambda. \end{aligned} \tag{B.3}$$

The terms in the first line of (B.2) represents the chemical potential of each vortex generated by ϕ_0 and ϕ_1 fluctuations, and the second line expresses the vortex-vortex interaction due to them. As has been reviewed in subsect.3.1, the interaction ($D(x;M_1)$) due to ϕ_1 is short-ranged and decays exponentially outside the range of $\mathrm{O}(Z_1^{-1})$. On the other hand, from (B.2) the probability to find a vortex on a

given position reads

$P = \exp(-2c_0\pi$

Hence, the average number of other

around a given vortex is evaluated to

$n_1^* \sim \exp(-2c_0\pi^2\kappa_0$

For example, in the limit: (i) $Z_1 \to 0$

both P and n_1^* vanish and the interacti

in the large-N and the very weak but

$Z_1 \, \frac{N-}{}$

with $\hat{e} \equiv e/\Lambda$ being a dimensionless g

by that of a massless scalar field (ϕ_0

potential

$$\begin{aligned} Z &= \prod_x \int \mathcal{D}\phi_0(x) \sum_n \exp\Big\{- \\ &\quad + \ln y_0 \sum_{x_0} n^2(x_0) \Big\}, \end{aligned}$$

This is equivalent to the massless s-

where the perturbative RG analysis c

transition to occur at the critical poin

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Figure Captions

Fig.1: A σ (a dashed line) tadpole
equation of $1/N$ expansion, which incl
are fermion propagators and the wavy
NG (π) boson.

Fig.2: ρ (dashed lines) tadpole d
equation of $1/N$ expansion, which co
lines are fermion propagators and the
a massless KT (χ) boson. Infrared c
cancelled with each other.

Fig.3: ρ (dashed lines) tadpole dia
The solid line is a fermion propagator

Fig.4: Feynman diagrams which co
tion in the leading order of $1/N$ expans
The wavy and dotted lines represent t

Fig.5: The RG flows for the ma
(X, Y_1) plane. The dotted line is a ph

Fig.6(a): The RG behaviors of Y
a scale parameter ℓ, in the small-X (
system.

Fig.6(b): The RG behaviors of Y
a scale parameter ℓ, in the large-X (
system.

Fig.7: The RG behaviors of $Y_1(\ell)$
large-X ($X(0) = 1.0$) regime of the ma
($Z(0) = 0.01$, 0.1: dashed and dotted

Fig.8(a): The RG behaviors of 1
as functions of a scale parameter ℓ, i

massive ($Z(0) = 0.01$) s-G system.

Fig.8(b): The RG scaling behaviors of $Y_1(\ell), Y_2(\ell), Y_3(\ell)$ and $Z(\ell)$ as functions of a scale parameter ℓ, plotted by a logarithmic scale in the vertical axis. Initial conditions are the same as those for fig.8(a).

Fig.9: The RG flows for the two-component massless s-G system ($b = 1$), projected on a (X, Y_1) plane. The dotted line is a phase boundary.

Fig.10: The RG flows for the coupled s-G system ($b = 1$) with $Z_1(0) = 0.1$, projected on a (X, Y_1) plane. The dotted line is a phase boundary.

Fig.11(a): The RG behaviors of $Y_1(\ell), 10^2 Y_2(\ell)$ and $10^2 W(\ell)$ as functions of a scale parameter ℓ, in the small-X ($X(0) = -0.02$) regime of the coupled s-G system ($b = 1$) with $Z_1(0) = 0.1$.

Fig.11(b): The RG behaviors of $Y_1(\ell), 10^3 Y_2(\ell)$ and $10^2 W(\ell)$ as functions of a scale parameter ℓ, in the large-X ($X(0) = 0.05$) regime of the coupled s-G system ($b = 1$) with $Z_1(0) = 0.1$.

Fig.12(a): The schematic behavior of the expected RG flows in the generalized scalar-vortex system with $b = 0$. The vertical line represents y and the horizontal line stands for other parameters $(\kappa, \beta, ...)$ collectively.

Fig.12(a): The schematic behavior of the expected RG flows in the generalized scalar-vortex system with $b > 0$. The vertical line represents y and the horizontal line stands for other parameters $(\kappa_j, \beta, ...)$ collectively.